

# UKF controlled Variable-Structure IMM Algorithms using Coordinated Turn Models

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**Abstract** – A new unscented Kalman filter controlled variable-structure- interacting multiple model technique is developed in this paper. The model set of this multiple model algorithm is built by different coordinated turn models. The corresponding turn rates and the state estimation process are realized by unscented Kalman filtering and ensures significant self-adjusting capabilities. The algorithm is distinguished through its highly accurate course and speed estimations for manoeuvring targets. This approach is compared with the variable-structure augmented interacting multiple model algorithm (VS-AIMM) defined by E. Semerdjiev, L. Mihaylova and X. Rong Li.

**Keywords:** IMM, estimation theory, control theory, target tracking, coordinated turn model, unscented transformation, filtering, EKF, UKF.

## 1 Introduction

The Interacting Multiple Model (IMM) is a very efficient estimation technique under unsure target manoeuvre hypotheses. Therefore the possible target manoeuvres are described by a finite discrete manoeuvre model set. The IMM is able to give very accurate estimation results, whenever the true target manoeuvre fits with a single manoeuvre model of the implemented discrete set. The additional gain of the IMM is to extend the covered target manoeuvres by a statistical mixing of the elements of the model set. A widely used model set contains the constant velocity model (CV) and the so called coordinated turn models (CT) for fixed angular velocity. For civil aircraft the assumption of constant turn rate is very applicable. However for military applications it is not adequate enough. In this case the turn rate is changed continuously. For example one has to take into account the weaving approach of an air to surface missile. To cover the variations of turn rate one has to enlarge the standard IMM algorithm by the usage of multiple coordinated turn models of different discrete turn rates. This has the disadvantage that the CPU load is increased by the number of models that are taken into account. Another possibility to extend the manoeuvre spectrum of an IMM algorithm is the augmentation. This technique substitutes discrete coordinated turn models by augmented coordinated turn (ACT) models. This is realized by extension of the state space by the turn rate [1]. A further extension is the combination of the augmentation idea

with the variable-structure extension of IMM algorithm, called VS-IMM. Under a variable-structure one understands the real-time modification of the model set [2]. Combining both techniques, the augmentation and the variable structure concept, one obtains the so called variable-structure augmented IMM algorithms, abbreviated by VS-AIMM. In [3] such a VS-AIMM implementation is described for a model set consisting of a right and left handed augmented coordinated turn model and an additional constant velocity model. However the filter concept used in [3] depends on the extended Kalman filter (EKF). Unfortunately the part of filtering, which considers the turn rate variations is a non-linear problem. To handle non-linear estimation and control problems, the unscented Kalman filtering (UKF) is very suited [4, 5]. Therefore the approach of this paper uses a UKF to realize both the control of the unknown turn rate and the state estimation.

A brief review of the dynamic systems and the IMM model is given in the second section. This is done to find a consistent nomenclature. Section 3 describes the model set which is used for the VS-AIMM and for the UKF controlled VS-IMM. Section 4 introduces the VS-AIMM and section 5 develops the new UKF controlled VS-IMM. The paper is finalized by a performance evaluation of the two algorithms by an appropriate simulation.

## 2 Dynamic systems and IMM Algorithms

The Interacting Multiple Model approach is a versatile technique to estimate a target state when the target manoeuvre is unsure and is subject to changes. To cover the possible manoeuvre spectrum, the IMM includes a finite set  $M$  of different manoeuvre models. Each Model  $m \in M$  describes a different dynamical system, which is expressed by a propagation and measurement equation. The propagation within the model  $m$  satisfies:

$$u_k^m = f^m(u_{k-1}^m) + q_{k-1}^m \quad (1)$$

Here  $u_k^m$  stands for the target state vector,  $q_{k-1}^m$  describes a Gaussian process noise of covariance  $Q_{k-1}^m$ . The

function  $f^m$  defines the target propagation which may depend on a control parameter  $\omega_{k-1}^m$ . The propagated state may be measured by the sensor measurement  $z_k$ . This measurement is related to the actual target state by

$$z_k = h(u_k^m) + r_k \quad (2)$$

with a measurement noise  $r_k$  defined as Gaussian distributed with covariance  $R_k$ .

The IMM algorithm uses four processing steps: interaction, mode-individual filtering, probability calculation and combination.

## 2.1 Interaction

For given target states  $u_{k-1}^m = u_{k-1|k-1}^m$  with corresponding covariances  $P_{k-1}^m = P_{k-1|k-1}^m$  and mixing probabilities  $\mu_{k-1|k-1}^{mn}$  for every model  $m \in M$ . One defines an initial estimate – covariance pair:

$$u_{k-1|k-1}^{0n} = \sum_{m \in M} u_{k-1|k-1}^m \mu_{k-1|k-1}^{mn} \quad (3)$$

$$P_{k-1|k-1}^{0n} = \sum_{m \in M} \mu_{k-1|k-1}^{mn} \left[ P_{k-1|k-1}^m + (u_{k-1|k-1}^m - u_{k-1|k-1}^{0n}) (u_{k-1|k-1}^m - u_{k-1|k-1}^{0n})^T \right] \quad (4)$$

## 2.2 Mode-individual filtering

This step performs for each model an individual filtering. Therefore Kalman or extended Kalman filtering is applied see e.g. [6, 7, 8]:

### 2.2.1 State and covariance propagation

The states and covariances are propagated to the time of the current measurement:

$$u_{k|k-1}^n = f^n(u_{k-1|k-1}^{0n}) \quad (5)$$

$$P_{k|k-1}^n = F_{k-1}^n P_{k-1|k-1}^{0n} (F_{k-1}^n)^T + Q_{k-1}^n \quad (6)$$

with

$$F_{k-1}^n = \frac{\partial f^n}{\partial u} \Big|_{u_{k-1}} \quad (7)$$

### 2.2.2 Measurement prediction and innovation covariance

One projects the states into the measurement space:

$$z_{k|k-1}^n = h(u_{k|k-1}^n) \quad (8)$$

$$S_k^n = H_k^n P_{k|k-1}^n (H_k^n)^T + R_k^n \quad (9)$$

### 2.2.3 Measurement residual and filter gain

The residuals and filter gains are calculated for each model:

$$v_k^n = z_k - z_{k|k-1}^n \quad (10)$$

$$K_k^n = P_k^n H_k^n (S_k^n)^{-1} \quad (11)$$

### 2.2.4 Update of the state estimator and covariance

$$u_{k|k}^n = u_{k|k-1}^n + K_k^n v_k^n \quad (12)$$

$$P_{k|k}^n = P_{k|k-1}^n - K_k^n S_k^n (K_k^n)^T \quad (13)$$

and the likelihood function

$$\Lambda_k^n = N(z_k; z_{k|k-1}^n, S_k^n) = \frac{1}{\sqrt{\det(2\pi S)}} \exp \left[ -\frac{1}{2} (z_k - z_{k|k-1}^n) (S_k^n)^{-1} (z_k - z_{k|k-1}^n)^T \right] \quad (14)$$

## 2.3 Probability evaluation

Let  $p^{n|m}$  be the transition probabilities between the different models. One calculate the mixing and mode probabilities:

### 2.3.1 Mixing probabilities

$$c_k^n = \sum_{m \in M} p^{n|m} \mu_{k-1}^m \quad (15)$$

$$\mu_{k-1|k-1}^{n|m} = \frac{p^{n|m} \mu_{k-1}^m}{c_k^n} \quad (16)$$

### 2.3.2 Mode probabilities

$$d_k = \sum_{m \in M} \Lambda_k^m c_k^m \quad (17)$$

$$\mu_k^n = \frac{\Lambda_k^n c_k^n}{d_k} \quad (18)$$

## 2.4 Combination

Finally the model-individual estimates and covariances are combined to an overall state and covariance. This is only done for output purpose and for further usage in data association algorithms.

$$u_{k|k} = \sum_{m \in M} u_{k|k}^m \mu_k^m \quad (19)$$

and

$$P_{k|k} = \sum_{m \in M} \mu_k^m \left[ P_{k|k}^m + (u_{k|k}^m - u_{k|k}) (u_{k|k}^m - u_{k|k})^T \right] \quad (20)$$

For more details on IMM, see [6, 7, 9].

## 3 Model set

For the model set  $M$ , the individual models have to be defined. One distinguished between (normal) constant velocity (CV) and coordinate turn models (CT) on the one

side and augmented models, i.e. augmented constant velocity (ACV) and augmented coordinated turn model (ACT) on the other side. The first models live in a four dimensional state space, defined by x- and y-position and x- and y-velocity. The augmented state consists again of x and y position and velocity, and an additional control parameter (turn rate) called  $\omega$ . A augmented state will be named by  $\hat{u}$ . For all models the measurement space is assumed 3-dimensional, given range, azimuth and range rate (=Doppler).

### 3.1 Constant velocity model (CV)

The constant velocity model is defined by the propagation:

$$u_k = f^{CV}(u_{k-1}) + q_{k-1} \quad (21)$$

The linear function  $f^{CV}$  defines the straight, not accelerated target propagation. As linear function it is expressed by matrix multiplication with

$$F^{CV} = \begin{pmatrix} 1 & 0 & t & 0 \\ 0 & 1 & 0 & t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (22)$$

The relationship between state and measurement space is given by

$$z_k = h(u_k) + r_k \quad (23)$$

i.e.

$$h\left(\begin{pmatrix} x & y & v_x & v_y \end{pmatrix}^T\right) = \begin{pmatrix} \sqrt{x^2 + y^2} \\ \arctan \frac{y}{x} \\ \frac{xv_x + yv_y}{\sqrt{x^2 + y^2}} \end{pmatrix} \quad (24)$$

The derivative is given by the linear map which is represented by the matrix

$$H = \frac{1}{\sqrt{x^2 + y^2}} \begin{pmatrix} x & y & 0 & 0 \\ -\frac{y}{\sqrt{x^2 + y^2}} & \frac{x}{\sqrt{x^2 + y^2}} & 0 & 0 \\ \frac{y^2 v_x - x y v_y}{x^2 + y^2} & \frac{x^2 v_y - x y v_x}{x^2 + y^2} & x & y \end{pmatrix} \quad (25)$$

### 3.2 Coordinated turn model (CT)

The coordinated turn model with turn rate  $\omega$  is then defined by the propagation mapping:

$$f^{CT}(u, \omega) = F^{CT}(\omega)u \quad (25)$$

$F^{CT}(\omega)$  is a linear mapping defined by

$$F^{CT}(\omega) = \begin{pmatrix} 1 & 0 & \frac{\sin \omega t}{\omega} & \frac{\cos \omega t - 1}{\omega} \\ 0 & 1 & \frac{1 - \cos \omega t}{\omega} & \frac{\sin \omega t}{\omega} \\ 0 & 0 & \cos \omega t & -\sin \omega t \\ 0 & 0 & \sin \omega t & \cos \omega t \end{pmatrix} \quad (26)$$

### 3.3 Augmented Constant velocity model (ACV)

The augmented constant velocity model is defined by the linear propagation  $f^{ACV}$  which may be expressed through matrix multiplication with

$$F^{ACV} = \begin{pmatrix} 1 & 0 & t & 0 & 0 \\ 0 & 1 & 0 & t & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (27)$$

The relationship between augmented state and measurement space is given by composition of  $h$  with the projection onto the state space without augmentation:

$$\hat{h}(\hat{u}) = h(u). \quad (28)$$

### 3.4 Augmented coordinated turn model (ACT)

Here the propagation is described by

$$f^{ACT}(\hat{u}) = \begin{pmatrix} f^{CT}(u, \omega) \\ \omega \end{pmatrix}, \quad (29)$$

i.e. the dynamical system is defined by

$$\hat{u}_k = \begin{pmatrix} f^{CT}(u_{k-1}, \omega_k) + q_{k-1} \\ \omega_{k-1} + w_{k-1} \end{pmatrix} \quad (30)$$

Again  $q_{k-1}$  describes a Gaussian process noise of covariance  $Q_{k-1}$  and  $w_{k-1}$  a Gaussian noise, which models the target manoeuvrability. The derivative of  $f^{ACT}$  is given by [1]:

$$F^{ACT} = \begin{pmatrix} 1 & 0 & \frac{\sin \omega t}{\omega} & \frac{\cos \omega t - 1}{\omega} & f_{15}^{ACT} \\ 0 & 1 & \frac{1 - \cos \omega t}{\omega} & \frac{\sin \omega t}{\omega} & f_{25}^{ACT} \\ 0 & 0 & \cos \omega t & -\sin \omega t & f_{35}^{ACT} \\ 0 & 0 & \sin \omega t & \cos \omega t & f_{45}^{ACT} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (31)$$

with

$$f_{15}^{ACT} = \frac{v_x(\omega t \cos \omega t - \sin \omega t) - v_y(\omega t \sin \omega t + \cos \omega t - 1)}{\omega^2} \quad (32)$$

$$f_{25}^{ACT} = \frac{v_x(\omega t \sin \omega t + \cos \omega t - 1) - v_y(\omega t \cos \omega t - \sin \omega t)}{\omega^2} \quad (33)$$

$$f_{35}^{ACT} = -t(v_x \sin \omega t + v_y \cos \omega t) \quad (34)$$

$$f_{45}^{ACT} = t(v_x \cos \omega t - v_y \sin \omega t) \quad (35)$$

#### 4 VS-AIMM Algorithm with coordinated turn models

The VS-AIMM algorithm [3] is now easy to explain. First one has to select a model set. The VS-AIMM in [3] uses a left handed and a right handed augmented coordinated turn model together with an augmented constant velocity model, i.e.:

$$M = \{ACT\_left, ACV, ACT\_right\} \quad (36)$$

Having defined these models, one must ensure, that the models are kept separated. Otherwise the turn models could become identical to the ACV model, whenever the turn rate converges to 0. Therefore one introduces boundaries for the turn rates  $\omega^{left}$  and  $\omega^{right}$  of the ACT\_left and ACT\_right model and requires the turn rates to kept within these boundaries, i.e.

$$\omega^{left} \in [\omega_{min}^{left}, \omega_{max}^{left}] \quad (37)$$

and

$$\omega^{right} \in [\omega_{min}^{right}, \omega_{max}^{right}] \quad (38)$$

with

$$\omega_{min}^{left} < \omega_{max}^{left} < 0 < \omega_{min}^{right} < \omega_{max}^{right} . \quad (39)$$

These boundaries complete the modification. The VS-AIMM algorithm applies the IMM technique with above model set on the augmented states. Before the individual (augmented) filtering steps the turn rates are set to the boundaries whenever necessary.

#### 5 UKF controlled VS-IMM

The new UKF controlled VS-IMM contains the right and left handed coordinated turn model together with a constant velocity model in its model set, i.e.

$$M = \{CT\_left, CV, CT\_right\} \quad (40)$$

Instead of the augmentation as in the VS-AIMM one estimates the parameters through a separate UKF

controller. Having determined the turn rate  $\omega_k^{left}$ , resp.  $\omega_k^{right}$  one has to ensure the separability of the models. Therefore one aligns the turn rates with predefined boundaries, as in the VS-AIMM, such that

$$\omega_k^{left} \in [\omega_{min}^{left}, \omega_{max}^{left}] , \quad (41)$$

$$\omega_k^{right} \in [\omega_{min}^{right}, \omega_{max}^{right}] \quad (42)$$

for predefined boundaries

$$\omega_{min}^{left} < \omega_{max}^{left} < 0 < \omega_{min}^{right} < \omega_{max}^{right} . \quad (43)$$

It follows the individual state estimation for the left-handed, constant velocity and right handed turn models. The mixing with respect to the overall IMM is done only with respect to the ordinary state  $u$ . This is an essential advantage of this approach which is therefore more flexible for adding of further manoeuvre models.

The following two subsections describe the UKF controller for the turn rates and the state estimation processes for each individual model.

##### 5.1 UKF controller for turn rate

The estimation of the turn rate depends on the control relation given by

$$\omega_k^m = \omega_{k-1}^m + \delta_{k-1}^m , \quad (44)$$

where  $\delta_{k-1}^m$  is assumed to be a Gaussian distributed noise of covariance  $\Delta_{k-1}^m$ . Further the state (1) and measurement equations (2) are combined to define a measurement relationship for the control-step:

$$z_k = h \circ f^m(u_{k-1}^m, \omega_{k-1}^m) + R_{k-1}^m \quad (45)$$

These equations define the foundation for the turn rate estimation via an UKF:

One starts with the calculation of sigma points for the turn rate. Define (see [2])

$$\sigma_0 = \frac{8}{3}, \sigma_1 = \sigma_2 = \frac{1}{6} \quad (46)$$

and

$$\tau_0 = \frac{2}{3}, \tau_1 = \tau_2 = \frac{1}{6} . \quad (47)$$

The so called sigma points are defined by

$$\omega_{k-1,0}^m = \omega_{k-1}^m , \quad (48)$$

$$\omega_{k-1,1}^m = \omega_{k-1}^m + \sqrt{3\Omega_{k-1}^m} , \quad (49)$$

$$\omega_{k-1,2}^m = \omega_{k-1}^m - \sqrt{3\Omega_{k-1}^m} \quad (50)$$

### 5.1.1 Parameter and parameter covariance propagation

It follows the propagation of the parameter to the current time:

$$\omega_{k|k-1}^m = \sum_{i=0}^2 \sigma_i \omega_{k-1,i}^m = \omega_{k-1}^m \quad (51)$$

$$\begin{aligned} \Omega_{k|k-1}^m &= \sum_{i=0}^2 \tau_i (\omega_{k-1,i}^m - \omega_{k|k-1}^m) (\omega_{k-1,i}^m - \omega_{k|k-1}^m)^T + \Delta_{k-1}^m \\ &= \Omega_{k-1}^m + \Delta_{k-1}^m \end{aligned} \quad (52)$$

### 5.1.2 Measurement prediction and parameter innovation covariance

$$\phi_{k|k-1,i}^m = h \circ f^m(u_{k-1,i}^m, \omega_{k-1,i}^m) \quad (53)$$

$$\phi_{k|k-1}^m = \sum_{i=0}^2 \tau_i \phi_{k|k-1,i}^m \quad (54)$$

$$\begin{aligned} \Phi_{k|k-1}^m &= \sum_{i=0}^2 \sigma_i (\phi_{k|k-1,i}^m - \phi_{k|k-1}^m) (\phi_{k|k-1,i}^m - \phi_{k|k-1}^m)^T + R_k \\ \tilde{\Phi}_{k|k-1}^m &= \sum_{i=0}^2 \sigma_i (\omega_{k|k-1,i}^m - \omega_{k|k-1}^m) (\phi_{k|k-1,i}^m - \phi_{k|k-1}^m)^T \end{aligned} \quad (55)$$

$$\begin{aligned} \tilde{\Phi}_{k|k-1}^m &= \sum_{i=0}^2 \sigma_i (\omega_{k|k-1,i}^m - \omega_{k|k-1}^m) (\phi_{k|k-1,i}^m - \phi_{k|k-1}^m)^T \\ &= \sum_{i=0}^2 \sigma_i (\omega_{k|k-1,i}^m - \omega_{k|k-1}^m) (\phi_{k|k-1,i}^m - \phi_{k|k-1}^m)^T \end{aligned} \quad (56)$$

### 5.1.3 Measurement residual and parameter filter gain

This step is analogue to the Kalman gain calculation.

$$\nu_{k|k-1}^m = \phi_{k|k-1}^m - z_k \quad (57)$$

$$\Gamma_{k|k-1}^m = \tilde{\Phi}_{k|k-1}^m (\Phi_{k|k-1}^m)^{-1} \quad (58)$$

### 5.1.4 Update of the parameter and the parameter covariance

$$\omega_{k|k}^m = \omega_{k|k-1}^m - \Gamma_{k|k-1}^m \nu_{k|k-1}^m \quad (59)$$

$$\Omega_{k|k}^m = \Omega_{k|k-1}^m - \Gamma_{k|k-1}^m (\Phi_{k|k-1}^m) (\Gamma_{k|k-1}^m)^T \quad (60)$$

## 5.2 State estimation

The turn rate estimations  $\omega_k^{left}$ , resp.  $\omega_k^{right}$  are adapted with the predefined boundaries. The result is again identified by the same symbols  $\omega_k^{left}$  and  $\omega_k^{right}$ .

Now one start with the state estimation based on above turn rate estimations. This uses the prediction equation

$$u_k^m = f^m(u_{k-1}^m, \omega_k^m) + q_{k-1}^m \quad (61)$$

and the measurement equation

$$z_k = h(u_k^m) + r_k \quad (62)$$

Again the sigma points are calculated after the setting :

$$\hat{\sigma}_0 = \frac{5}{3}, \hat{\sigma}_1 = \dots = \hat{\sigma}_8 = \frac{1}{6} \quad (63)$$

and

$$\hat{\tau}_0 = -\frac{1}{3}, \hat{\tau}_1 = \dots = \hat{\tau}_8 = \frac{1}{6}. \quad (64)$$

One defines the sigma points:

$$u_{k-1,0}^m = u_{k-1}^m, \quad (65)$$

$$u_{k-1,i}^m = u_{k-1}^m + \left( \sqrt{3P_{k-1}^m} \right)_i, \quad (66)$$

$$u_{k-1,i+4}^m = u_{k-1}^m - \left( \sqrt{3P_{k-1}^m} \right)_i \quad (67)$$

### 5.2.1 State and covariance propagation

$$u_{k|k-1,i}^m = f^m(u_{k,i}^m, \omega_k^m) \quad (68)$$

$$u_{k|k-1}^m = \sum_{i=0}^8 \hat{\sigma}_i u_{k|k-1,i}^m \quad (69)$$

$$\begin{aligned} P_{k|k-1}^m &= \sum_{i=0}^8 \hat{\tau}_i (u_{k|k-1,i}^m - u_{k|k-1}^m) (u_{k|k-1,i}^m - u_{k|k-1}^m)^T + Q_{k-1}^m \\ &= \sum_{i=0}^8 \hat{\tau}_i (u_{k|k-1,i}^m - u_{k|k-1}^m) (u_{k|k-1,i}^m - u_{k|k-1}^m)^T + Q_{k-1}^m \end{aligned} \quad (70)$$

### 5.2.2 Measurement prediction and innovation covariance

$$z_{k|k-1,i}^m = h(u_{k|k-1,i}^m) \quad (71)$$

$$z_{k|k-1}^m = \sum_{i=0}^8 \hat{\sigma}_i z_{k|k-1,i}^m \quad (72)$$

$$\begin{aligned} P_{k|k-1}^m &= \sum_{i=0}^8 \hat{\tau}_i (z_{k|k-1,i}^m - z_{k|k-1}^m) (z_{k|k-1,i}^m - z_{k|k-1}^m)^T + R_k \\ &= \sum_{i=0}^8 \hat{\tau}_i (z_{k|k-1,i}^m - z_{k|k-1}^m) (z_{k|k-1,i}^m - z_{k|k-1}^m)^T + R_k \end{aligned} \quad (73)$$

$$\tilde{P}_{k|k-1}^m = \left( \sum_{i=0}^8 \hat{\tau}_i (u_{k-1,i}^m - u_{k-1}^m) (z_{k|k-1,i}^m - z_{k|k-1}^m)^T \right) \quad (74)$$

### 5.2.3 Measurement residual and filter gain

$$\nu_{k|k-1}^m = z_{k|k-1}^m - z_k \quad (75)$$

$$K_{k|k-1}^m = \tilde{P}_{k|k-1}^m (P_{k|k-1}^m)^{-1} \quad (76)$$

### 5.2.4 Update of the parameter state and parameter covariance

$$u_{k|k}^m = u_{k|k-1}^m - K_{k|k-1}^m (z_{k|k-1}^m - z_k) \quad (77)$$

$$P_{k|k}^m = P_{k|k-1}^m - K_{k|k-1}^m (P_{k|k-1}^m) (K_{k|k-1}^m)^T \quad (78)$$

## 6 Performance Evaluation

This section demonstrates the performance of the VS-AIMM and UKF-controlled VS-IMM via a simulation example.

### 6.1 Scenario definition

The target profile consists of four  $180^\circ$  turns connected via straight segments. The turns possess a cross-acceleration of 1, 2, 3, and 4 g. The corresponding turn rates are 0.05 rad/s, 0.1 rad/s, 0.15 rad/s, and 0.2 rad/s. The target speed was 200 m/s and the scenario was performed in a distance between 20 km and 36.6 km. The sensor was simulated with a range accuracy (standard deviation) of 10 m, azimuth accuracy of 10 mrad and Doppler accuracy of 5 m/s. Figure 2 shows a x-y-plot of the simulated sensor measurements of a single run.

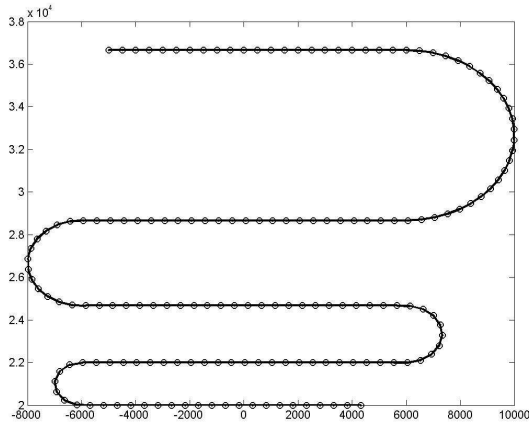


Fig. 1. True trajectory.

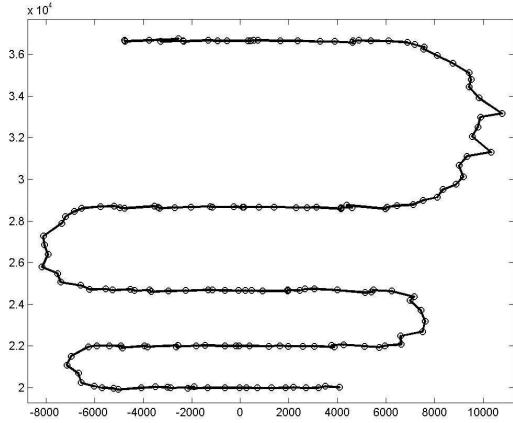


Fig. 2. Simulated sensor measurements.

### 6.2 UKF-controlled VS-IMM results

In the following the results of UKF-controlled VS-IMM are presented. Figure 3 shows the x-y-plot of a single run. Figures 4-8 show the model probabilities and the estimated turn rates as result of a Monte Carlo simulation.

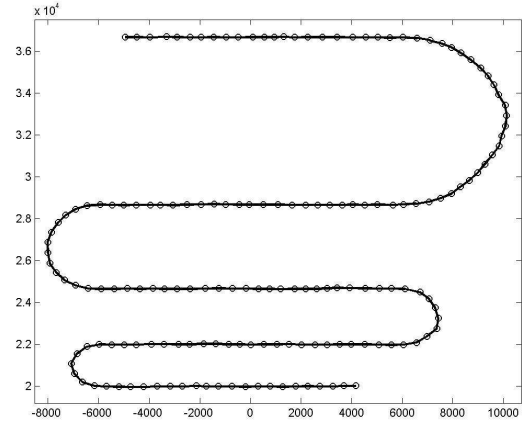


Fig. 3. x-y-plot of UKF-controlled VS-IMM.

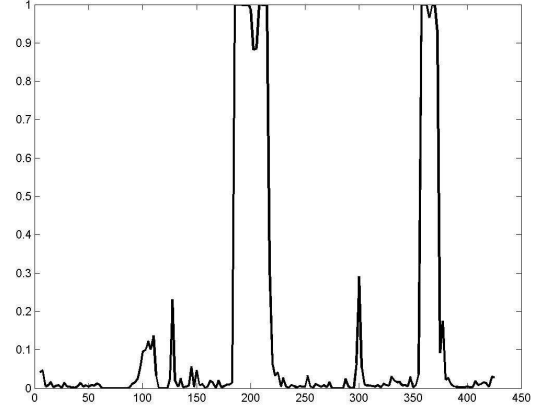


Fig. 4. UKF probability for left-handed turn.

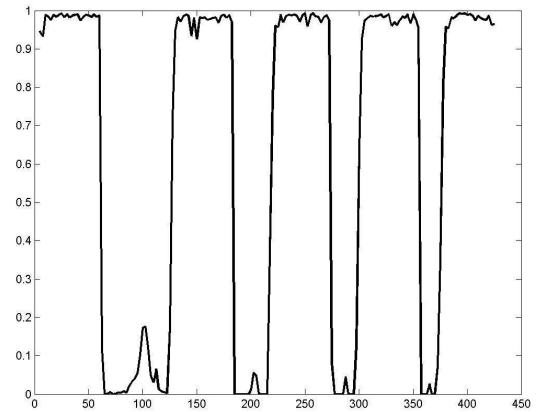


Fig. 5. UKF probability for constant velocity model.

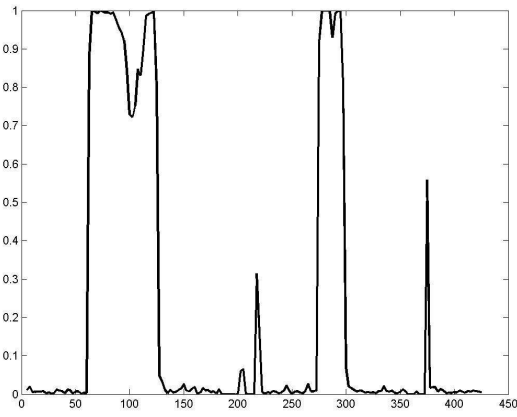


Fig. 6. UKF probability for right handed turn model.

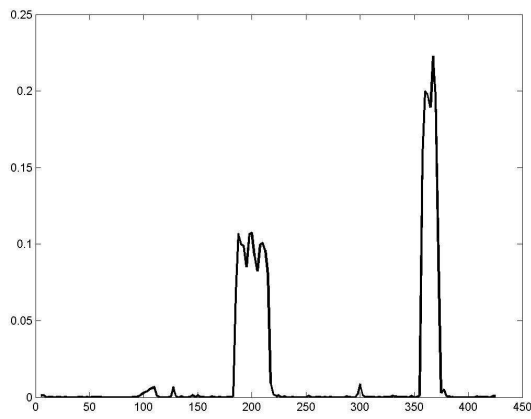


Fig. 7. UKF estimated left handed turn rates.

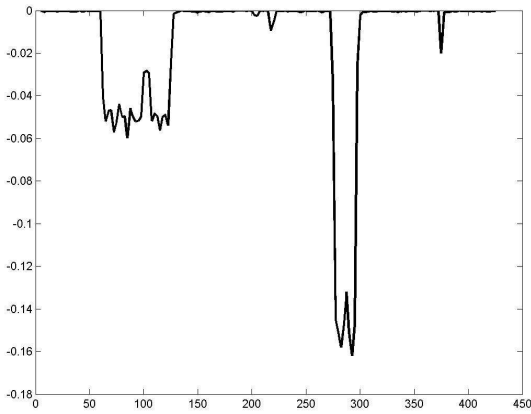


Fig. 8. UKF estimated right handed turn rate.

### 6.3 VS\_AIMM results

The following pictures show the result of the VS-AIMM with EKF based filtering techniques. Again one x-y-plot, and the model probabilities and estimates turn rates are shown.

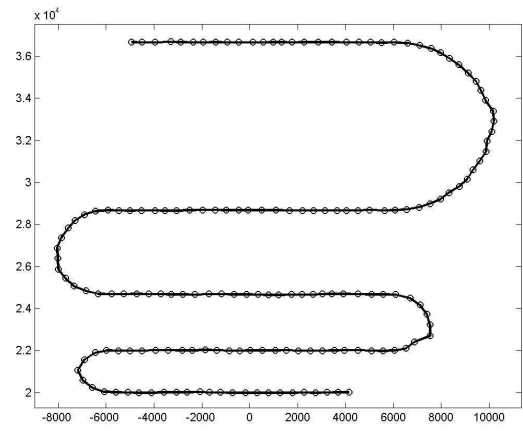


Fig. 9. VS-AIMM x-y-plot.

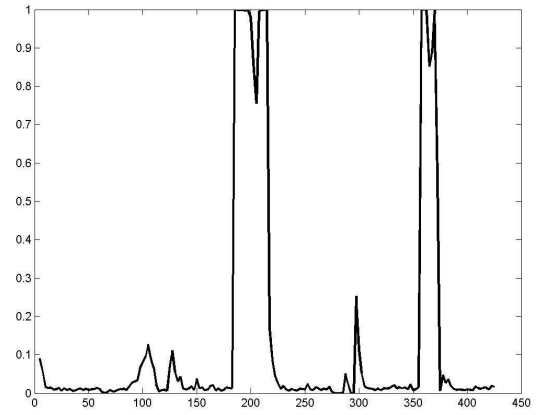


Fig. 10. VS-AIMM left-hand turn probabilities.

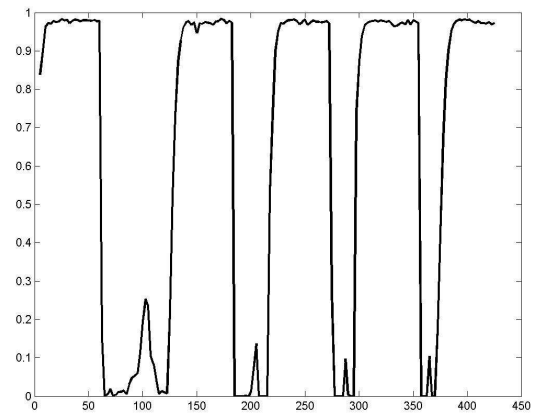


Fig. 11. VS-AIMM constant velocity probability.

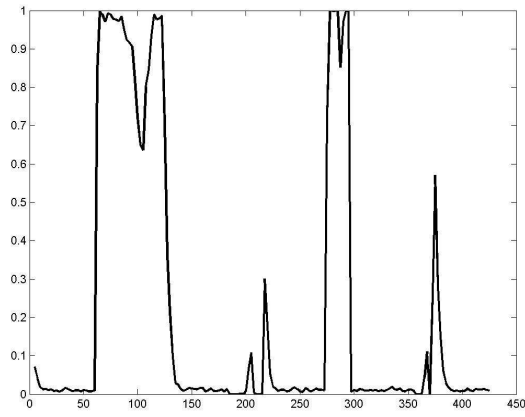


Fig. 12. VS-AIMM right-handed turn probability.

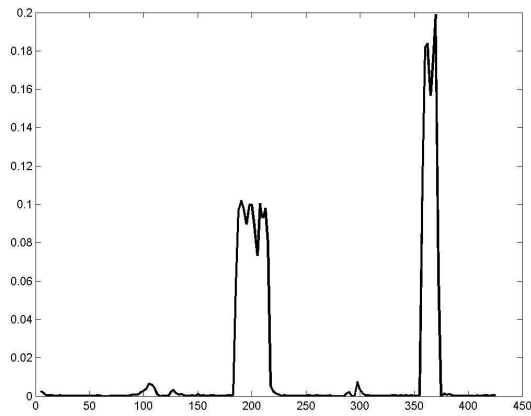


Fig. 13. VS-AIMM left handed turn rates.

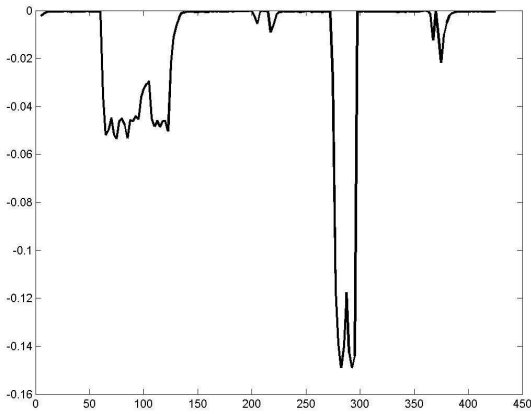


Fig. 14. VS-AIMM right handed turn rates.

## 6.4 Summary

The table 1 below gives a summary of the

- range,
- azimuth,
- course, and
- speed

deviation over the whole trajectories through Monte Carlo runs.

Table 1. Comparison of performance

	VS-AIMM	UKF controlled VS-IMM
range	6.4 m	6.5 m
azimuth	0.003 rad	0.003 rad
speed	4.1 m/s	3.6 m/s
course	0.029 rad	0.019 rad

## 7 Conclusions

A new UKF controlled VS-IMM based on coordinated turn models was described. Both VS-AIMM and the new UKF-controlled VS-IMM are proved to possess excellent estimation accuracy. The UKF based approach improves the VS-AIMM with respect to course and speed accuracy. It has the further advantage to be more modular in the extension of the model set with additional models.

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